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Determination of Stock Option Prices using the Trinomial Hull and Black Scholes Methods on Apple Inc. Corporation

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ABSTRACT

The rapid development of asset buying and selling transactions makes investors want an investment that can minimize financial risks. This is the background for the introduction of derivative instruments. One of the widely used derivative products is stock options. There are various types of stock options used in the capital market, including European Options which consist of Call Options and Put Options. To determine the price of European options, several methods are used. Among them are the trinomial method and the Black Scholes formula. This study aims to calculate the price of European Call and Put options using the trinomial method and Black Sholes on Apple Inc. stocks with Matlab software. The results show that the trinomial method has a convergent nature to the Black Scholes model for both Call Options and European Put Options.

Keywords: Apple Inc., Black-Scholes, options, trinomial

1. INTRODUCTION

The capital market is an activity related to public offering and securities trading, public companies related to the securities it issues, as well as institutions and professions related to securities. Principal assets that can be traded in the capital market include stocks, bonds, stock indices, bond indices, currencies, interest rates, and other financial instruments. The rapid development of asset buying and selling transactions makes investors want an investment that can minimize financial risks. This is the background for the introduction of derivative instruments [1].

Derivative instruments can be defined as financial instruments in the form of agreements or contracts between two parties where the opportunity or profit is related to the price of other underlying assets. Derivative products that have been widely recognized include *futures contracts*, *forward contracts*, *swap*s, and *options* [1], [2]. According to Fabozzi and Peterson [3], options have advantages in handling financial risk because they can be used to determine the maximum and minimum limits of asset prices, so it is very useful to overcome the possibility of an increase or decrease in asset prices at a specified time.

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Option is the right, not the obligation, of the holder to buy or sell an asset at a specific price and time. Usually the underlying asset is stocks. Since this is a right, the holder can carry out the purchase, sale, or not. According to the situation. If the holder exercises his right, the issuer of the option is obliged to exercise it [4][5] . There are several types of options. One type of option that is often used is the European option, which consists of the European Call option and the European Put option [6].

Call Option is a type of contract that gives the option buyer the right to buy from the option seller a certain share at a certain price and timeframe. A Put option is an option that gives the holder the right to sell a certain number of shares of a particular company to the option seller at a specific price and time [7]. Based on the expiration time, options can be grouped into two, namely European-type options and American-type options. European type options are options that can be exercised at maturity only, while American type options are options that can be exercised at or before maturity [8] [9].

There are several models that have been used to determine the harfa of options. Among them are the Black Scholes model, the binomial model, the trinomial model, and others [10]. The Black Scholes formula was first introduced by Fisher Black and Myron in 1973 to determine the price of European-type options assuming the absence of dividend payments, the absence of transaction fees, constant risk-free interest rates, and changes in stock prices following random patterns [11]. Another model that is often used is the trinomial model. The trinomial model is a development of the Binomal model which assumes that the stock price at a certain point can move up, fixed or down. The trinomial model is convergent to the Black-Scholes model [12] . Several previous studies have used stock pricing methods for several different stock companies. Among them is Langat [11] who uses the Black Scholes and trinomial methods in his research. In addition, Lilyana [5]used these models to calculate the stock price of Microsoft companies, among others.

This study aims to determine the price of European Call Options and Put Options using the Hull trinomial method and compare them with the Black-Scholes formula. The stocks that will be used as the object of this study are the model of Apple Inc.

2. METHODS

The data used in this study is the daily stock price of Apple Inc. from July 24, 2023 to July 24, 2024 sourced from the www.yahoofinance.com website [13]. In addition, the risk free rate parameter was taken from the www.bi.go.id website on July 17, 2024 [14]. The first step is to determine the volatility of the stock price (σ), by following these steps [8] [15].

1. Calculating stock returns using equation (1)

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

   where S_t is the stock price at the time of t.

2. Determining expected return of stock using equation (2)

$$\bar{R}_t = \frac{1}{n}\sum_{t=0}^{n} R_t \tag{2}$$

3. Calculating stock price volatility (σ) using equation (3)

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=0}^{n}(R_t - \bar{R}_t)^2 h} \tag{3}$$

where h is the length of trading time for one year ($h = 252$)

Trinomial Method for European Options

The trinomial method is a development of the binomial method. If in the binomial method the movement of the stock price is assumed to have increased or decreased, then in the trinomial method the stock price is assumed to have increased, fixed, or decreased. The time interval [0, T] is divided into n uniform sub-intervals with a width of $\Delta t = \frac{T}{n}$. The dividing points consist of $0 = t_0 < t_1 < \cdots < t_n = T$, where $t_j = j\Delta t$ where $(j = 0, 1, 2, \dots, n)$ which meets the following conditions.

1. In a short interval of Δt, it is possible that the stock price can rise, fix, or fall. Suppose it is known that the stock price at the starting point is S, then the stock price will change to Su, S, or, Sd where u is the increase factor, and d is the decrease factor whose value is determined by the equation (4)

$$u = e^{\sigma\sqrt{3\Delta t}} \qquad d = e^{-\sigma\sqrt{3\Delta t}} \tag{4}$$

   where σ is the volatility of the stock price.
2. Determine the probability of going up, P_u, the probability of going down, P_d, by first choosing a fixed probability based on the Hull trinomial method, which is $P_m = 2/3$, where

$$pu = \frac{1}{6} + \left(r - \frac{1}{2}\sigma^2\right)\sqrt{\frac{\Delta t}{12\sigma^2}} \qquad pd = \frac{1}{6} - \left(r - \frac{1}{2}\sigma^2\right)\sqrt{\frac{\Delta t}{12\sigma^2}} \tag{5}$$

3. Determine the price of a stock in each period (S_{ij}) using equation (6).

$$S_{ij} = \begin{cases} u^i S_0 & for\ i \geq 1 \\ S_0 & for\ i = 0 \\ d^i S_0 & for\ \ i \leq -1 \end{cases} \tag{6}$$

4. Determining the payoff value for Call and Put options for European Options using the equation (7).

$$C_{iN} = max\left[\left(S_{ij} - K\ \right), 0\right] \tag{7}$$

   for Call option, and using the equation (7).

$$P_{iN} = max\left[\left(K - S_{ij}\ \right), 0\right] \tag{8}$$

for Put option.

5. Determining the price of Call and Put options using the equation (8)

$$C_{i,j} = e^{-r\Delta t}(p_u.C_{i+1,j+1} + p_m.C_{i+1,j+1} + p_u.C_{i+1,j+1}) \quad (9)$$

for Call option, and using the equation

$$P_{i,j} = e^{-r\Delta t}(p_u.P_{i+1,j+1} + p_m.P_{i+1,j+1} + p_u.P_{i+1,j+1}) \quad (10)$$

for Put option.

Black-Scholes Method for European Options

The Black-Scholes formula is a Partial Differential Equation (PDE) that estimates the price of a European Option over time. The determination of the solution to the Black-Scholes equation can be obtained in three ways, namely by using the assumption that stock prices are distributed lognormally, by solving the Black-Scholes differential equation, and the third by using the properties of the central limit theorem on the binomial model. By using one of the three methods, the Black-Scholes formula for the European Option will be obtained as in the equation (11)

$$C = S_0N(d_1) - Ke^{-rT}N(d_2) \quad (11)$$

for the Call option, and using the equation (12).

$$C = Ke^{-rT}N(-d_2) - S_0N(-d_1) \quad (12)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(\frac{r+\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(\frac{r-\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where T is the maturity time $\left(T = \frac{N}{h}\right)$, where N is many periods of stock trading before expiration date, $N(d_1)$ is cumulative distribution of d_1 and $N(d_2)$ is cumulative distribution of d_2. The model calculation in this study will use Ms. Excel and Matlab 2017b.

3. RESULTS AND DISCUSSION

The first step is to calculate the volatility of the stock price using equations (1) to (4). By using Ms. Excel, a volatility of $\sigma = 22.37\%$ was obtained. The initial stock price that is the reference (S0) in this study is the closing stock price on July 19, 2024, which is $S_0 = \$223.96$. The option contract used is a contract with the name strike price (K) that will be used is a contract with the AAPL240726C00160000 name issued on July 19, 2024 with a strike price, $K = \$160$. Meanwhile, the risk-free interest rate is taken from the BI-Rate interest rate on July 17, 2024, which is $r = 6.25\%$.

The next stage is to calculate the price of European Call and Put options using Matlab 2017b. As explained in the previous section, there are two methods of determining option

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pricing that will be used in this study, namely the Trinomial Hull model and the Black-Scholes formula. Figure 3.1 shows a binomial tree of stock prices using equation (6).

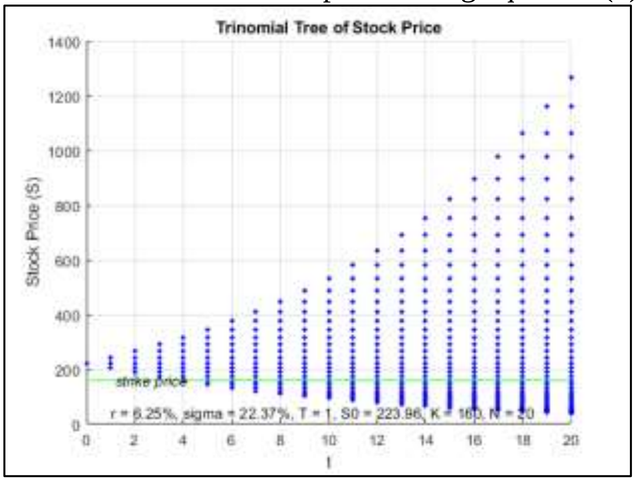

Figure 3.1. Trinomial Tree of Stock Price.

Based on the calculation results with Matlab2017b, the prices of European Call and Put options are obtained using the Trinomial and Black-Scholes methods as in Table 3.1.

Methods	Call Options	Put Options
Trinomial Hull	$74.2428	$0.59678
Black-Scholes	$74.2629	$0.60898

Table 3.1. European Call and Put Options Prices with Trinomial Hull and Black-Scholes.

Based on Table 3.1. above it can be seen that the price of European call and put options with the Trinomial Hull method converges to the Black-Scholes method, both for Call and Put options. This is in line with the results of Puspita's research [12], which proves that the trinomial model converges with the Black-Scholes model.

CONCLUSION

Based on the results and discussion, it can be concluded that the trinomial method can be used to calculate the price of European options. The simulation results show that the trinomial model is convergent towards the Black-Scholes model.

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